DISCRETE MATHEMATICS 1 — SOMMERSEMESTER 2012

email: person@math.fu-berlin.de

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Exam

Please state precisely every theorem from the lecture that you are going to apply.

Problem 1 []

(a) Draw the tree on the vertex set [10] with the following Prüfer code: $(6, 9, 2, 4, 2, 4, 9, 9)$.

(b) Draw all self-complementary (up to isomorphism) graphs on 4 and 5 vertices (and prove that all other graphs with 4 or 5 vertices are not self-complementary).

Problem 2 []

Prove the following identity:

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}\binom{m+n-i}{k-i} = \begin{cases}\binom{m}{k} & \text{if } m \geq k,\\ 0 & \text{if } m < k.\end{cases}$$

Problem 3 []

Prove that the chromatic number of a graph which has exactly one cycle of odd length is 3.

Problem 4 []

State the theorem of Hall about matchings in bipartite graphs and the theorem of Kőnig-Egerváry about matchings and covers in bipartite graphs. Further show that Kőnig-Egerváry theorem implies Hall's theorem.

Problem 5 []

Find an explicit formula (either using the generating functions or the characteristic polynomial) for the sequence (a_n) defined by the following recurrence relation:

$$a_{n+2} = 6a_{n+1} - 9a_n \quad \text{for} \quad n \geq 1,$$

with $a_1 = 1$ and $a_2 = 2$.

Problem 6 []

Let $d \in \mathbb{N}$ and $d \geq 2$.

(a) Show that in the d-dimensional cube Q_d for every vertex v there is a unique vertex $\bar{v}$ at distance d.

(b) Show that every spanning tree T in Q_d has diameter at least $2d - 1$. You may want to consider for every $v \in V(Q_d)$ the first edge on the v-$\bar{v}$-path in T.

(c) Find a spanning tree in Q_d of diameter exactly $2d - 1$.