

Mock Exam

Please state precisely every theorem from the lecture that you are going to apply
Try to write down the solutions within 2 hours, without any help of books etc.

Due July 9, after the lecture

Problem 1

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- (a) Define the chromatic number and chromatic index of a graph.
- (b) Let $n \geq 2$. Determine (find and prove) the chromatic number of the following graphs: K_n , T_n (any tree with n edges), $K_n \setminus M$ (where M is a maximum matching in K_n).
- (c) What is the chromatic index of a tree T_n ?

Problem 2

□

- (a) Define the Stirling numbers $S_{n,k}$ of the second kind.
- (b) State and prove the recursive formula for $S_{n,k}$.

Problem 3

□

Find the generating function for the the number h_n of baskets with n fruits consisting of apples, oranges, bananas and peaches, where in each basket there are: even number of apples, at most two oranges, $3k$ of bananas (for some $k \in \mathbb{N}_0$) and at most one peach. Find the formula of h_n from the generating function.

Problem 4

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Prove that the number of number-partitions of n into (any number of) distinct terms is equal to the number of number-partitions of n into odd terms.

Problem 5

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Formulate and prove the Theorem of Petersen about matchings in cubic bridgeless graphs.

Problem 6

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- (a) Define the Ramsey number $R(k, k)$.
- (b) Show that if the edges of K_{17} are colored with three colors then there is a monochromatic triangle in it.