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Example sheet 11

Due June 25, after the lecture

Problem 1 [to be submitted]
Let G be a bipartite regular graph. Show that G is 2-connected if and only if G is connected.

Problem 2 []
Prove that every graph G has a bipartite subgraph with at least $e(G)/2$ edges.

Problem 3 []
Let $G = (V, E)$ be a graph with $E \neq \emptyset$. Show that one can partition V into two nonempty sets V_1 and V_2 such that:

(a) $\chi(G[V_1]) + \chi(G[V_2]) = \chi(G)$ holds.

(b) $\chi(G[V_1]) + \chi(G[V_2]) > \chi(G)$ if G is not complete.

Problem 4 []

(a) Let $V_1 \dot\cup V_2 \dot\cup \ldots \dot\cup V_k = V$ be a partition of an arbitrary graph G such that for every pair $i \neq j \in [k]$ there exist vertices $x \in V_i$ and $y \in V_j$ such that $xy \notin E(G)$. Show that
$$\chi(G) \leq |G| - k + 1.$$

(b) Show that
$$\chi(G) + \chi(\overline{G}) \leq |G| + 1 \text{ and } \chi(G)\chi(\overline{G}) \geq |G|.$$

Problem 5 []

(a) Show that a graph on three or more vertices is 2-connected if and only if any two distinct vertices are connected by two paths with no common inner vertices.

(b) Let G be a 2-connected graph and let e_1, $e_2 \in E(G)$. Show that there is a cycle in G which contains both edges e_1 and e_2.

Problem 6 []
A *block* B of a graph G is a subgraph without cutvertices and is maximal with respect to this property. (Notice that B itself can contain cutvertices of G.) The block graph $B(G)$ of G is a bipartite graph with bipartition $\mathcal{B} \cup S$, where $\mathcal{B}$ is the set of blocks of G and S is the set of cutvertices of G, where a block B is adjacent to a cutvertex s if and only if B contains s. Show the following facts about $B(G)$:

(a) Any block is either a single vertex or a bridge or a maximal 2-connected subgraph of G.

(b) Two blocks intersect in at most one cutvertex (of G).

(c) $B(G)$ is a forest.

(d) $B(G)$ is a tree if and only if G is connected.

Further, calculate the chromatic number of G in terms of the chromatic numbers of its blocks.