Discrete Mathematics 1 | SommerSemester 2012
email: person@math.fu-berlin.de | 26 June 2012

Example sheet 12

Due Juli 3, after the lecture

Problem 1 [to be submitted]

Show that every graph can be embedded in $\mathbb{R}^3$ in such a way that:

(i) each vertex lies on the curve $\{(t, t^2 t^3) \colon t \in \mathbb{R}\}$, and

(ii) each edge is a straight line segment.

Problem 2 []

Show that neither $K_{3,3}$ nor K_5 are planar. Further show that the graphs formed by deleting one edge from K_5 and $K_{3,3}$ are planar.

Given a graph $G = (V, E)$ and an integer j define G^j, jth power of G, as follows:

$$G^j := (V, \{uv \colon \mathbf{dist_G(u,v)} \leq \mathbf{j}\}),$$

where $\mathbf{dist_G(u,v)}$ is the length of a shortest path connecting u and v.

Problem 3 []

Find a connected graph whose square G^2 is not hamiltonian.

Problem 4 []

Show that the third power G^3 of a connected graph on at least 3 vertices contains a hamiltonian cycle.

A directed graph D is a pair of (V, E) of the vertex set V and the edge set E with $E \subseteq [V]^2 \setminus \{(v, v) \colon v \in V\}$. An edge uv is directed from u to v. An orientation of a graph $G = (V, E)$ is a directed graph $D = (V, E')$ where every edge $e = \{u, v\}$ from E is replaced by an edge uv or vu. An oriented complete graph is called a tournament.

Problem 5 []

Show that every tournament contains a (directed) hamiltonian path. (A directed hamiltonian path is a sequence of vertices $v_1, \ldots, v_{|G|}$, such that $v_i v_{i+1}$ is an edge in the directed graph.)

Problem 6 []

Show that for a graph H there is a graph G such that $H = L(G)$ if and only if $E(H)$ can be partitioned into complete subgraphs, with each vertex of H appearing in at most two of these complete subgraphs.
Find a graph H which is not a line graph. Find two nonisomorphic graphs G_1 and G_2 whose line graphs are isomorphic.