Discrete Mathematics 1 — SommerSemester 2012

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Example sheet 2

Due April 23, after the lecture (10 a.m.)

Problem 1 [to be submitted]

The number of all partitions of an n-element set is the *Bell number* B_n. Thus, $B_n = \sum_{k=0}^{n} S_{n,k}$ (recall that $S_{n,k}$ are the Stirling numbers of the second kind).

(a) Show that

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

(b) Show the following formula for the Bell numbers ($n \in \mathbb{N}_0$):

$$B_n = \frac{1}{e} \sum_{i=0}^{\infty} \frac{i^n}{i!}$$

Problem 2 []

(a) Prove $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ for $n \geq m \geq k \geq 0$ by counting pairs of sets (A, B) in two ways, and deduce $\sum_{k=0}^{m} \binom{n}{k}\binom{n-k}{m-k} = 2^m \binom{n}{m}$.

(b) Use part (a) to show that $\binom{2n}{2k}\binom{2n-2k}{n-k}\binom{2k}{k} = \binom{2n}{n}\binom{n}{k}^2$ for $n \geq k \geq 0$.

Problem 3 []

How many k-element subsets S are there in the set $[n] := \{1, \ldots, n\}$, such that S does not contain any two consecutive numbers (i.e. $\{i, i+1\} \not\subseteq S$)?

Problem 4 []

There are k kinds of postcards, but only in a limited number each, there being a_i copies of the i^{th} one. What is the number of possible ways of sending them to n friends? (We may send more than one copy of the same postcard to the same person).

Problem 5 []

(a) The number of partitions of the number n into (a sum of) no more than r terms is equal to the number of partitions of n into any number of terms, each at most r.

(b) The number of partitions of a number n into exactly m terms is equal to the number of partitions of $n - m$ into no more than m terms. Find a similar identity involving the number of partitions of n into exactly m *distinct* terms.