

Example sheet 3

Due April 30, after the lecture

Problem 1

[to be submitted]

- (a) How many positive integers less than 1000 have no factor between 1 and 10?
- (b) How many permutations of the numbers 1, 2, ..., 10 exist that map no even number to itself?

Problem 2

□

Show that $\sum_{k=0}^n S_{n+1,k+1} x^k = (x+1)^n$, and use the polynomial method to prove $S_{n+1,k+1} = \sum_{i=0}^n \binom{n}{i} S_{i,k}$. Verify this last equality also by a combinatorial argument, and deduce again the Bell number recurrence:

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k.$$

Problem 3

□

Find two different proofs of the following statement. The number of number-partitions of n into (any number of) distinct terms is equal to the number of number-partitions of n into odd terms.

- (a) use inclusion-exclusion principle,
- (b) find a bijection.

Problem 4

□

Given n balls of different colors and k indistinguishable boxes. How many possibilities are there to put balls into boxes such that

- (a) every box contains at most one ball?
- (b) every box contains at least one ball?
- (c) there are no restrictions?

Problem 5

□

Prove the following identity between binomial coefficients by using the principle of inclusion and exclusion.

$$\sum_{i=0}^n (-1)^i \binom{n}{i} \binom{m+n-i}{k-i} = \begin{cases} \binom{m}{k} & \text{if } m \geq k, \\ 0 & \text{if } m < k. \end{cases}$$