email: person@math.fu-berlin.de
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Example sheet 3

Due April 30, after the lecture

Problem 1 [to be submitted]

(a) How many positive integers less than 1000 have no factor between 1 and 10?

(b) How many permutations of the numbers 1, 2, ..., 10 exist that map no even number to itself?

Problem 2 []

Show that $\sum_{k=0}^{n} S_{n+1,k+1} x^{\underline{k}} = (x+1)^n$, and use the polynomial method to prove $S_{n+1,k+1} = \sum_{i=0}^{n} \binom{n}{i} S_{i,k}$. Verify this last equality also by a combinatorial argument, and deduce again the Bell number recurrence:

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

Problem 3 []

Find two different proofs of the following statement. The number of number-partitions of n into (any number of) distinct terms is equal to the number of number-partitions of n into odd terms.

(a) use inclusion-exclusion principle,

(b) find a bijection.

Hints

(a): the number of number-partitions of n into odd terms is the number of number-partitions of n minus those number-partitions that contain at least one even term. How can one express the number of number-partitions of n into distinct terms?

(b): Let $\lambda = \lambda_1, \ldots, \lambda_k$ be a partition of n into distinct numbers. What happens if we write each $\lambda_i = a_i \cdot 2^t$ where a_i is odd?

Problem 4 []

Given n balls of different colors and k indistinguishable boxes. How many possibilities are there to put balls into boxes such that

(a) every box contains at most one ball?

(b) every box contains at least one ball?

(c) there are no restrictions?

Problem 5 []

Prove the following identity between binomial coefficients by using the principle of inclusion and exclusion.

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} \binom{m+n-i}{k-i} = \begin{cases} \binom{m}{k} & \text{if } m \geq k, \\ 0 & \text{if } m < k. \end{cases}$$

Hint:

Suppose you are given $m+n$ points, m being colored red and n blue...