Discrete Mathematics 1 — SommerSemester 2012
email: person@math.fu-berlin.de — 30 April 2012

Example sheet 4

Due May 7, after the lecture

Problem 1 [to be submitted]

(a) Prove that the product of some two of the following three numbers is nonnegative: $65^{1000} - 8^{2001} + 3^{177}$, $79^{1212} - 9^{2399} + 2^{2001}$ and $24^{4493} - 5^{8192} + 7^{1777}$.

(b) Show that among n guests at some party there are two who know the same number of the guests at that party.

(c) From the integers 1, 2, ..., 200, we choose 101 integers. Show that among the integers chosen there are two such that one of them is divisible by the other. Can you prove the same statement if we choose only 100 integers (if not, then give a counterexample)?

Problem 2 []

How many monic polynomials of degree n are there in $\mathbb{F}_p[x]$ that do not take on the value 0 for any $x \in \mathbb{F}_p$? Recall: $\mathbb{F}_p$ (also known as $\mathbb{Z}_p$) is the finite field with p elements and a monic polynomial f has 1 as its leading coefficient: $f(x) = x^n + \sum_{i=0}^{n-1} a_i x^i$.

Problem 3 []

Find (and prove) the closed formula for the following sum

$$\sum_{k=0}^{n} (-1)^k \binom{2n-k}{k} 2^{2n-2k},$$

by considering the colorings of $[2n]$ by red and blue such that if i is red then $i-1$ is not blue.

Problem 4 []

A child receives a pack of 45 candies. She eats at least one each day during the month of April. Prove that there is a period (of consecutive days) during which she eats exactly 14.

Problem 5 []

Show for every $n \in \mathbb{N}$ (and for the Euler function φ):

$$\sum_{d \in \mathbb{N},\, d | n} \varphi(d) = n.$$