email: person@math.fu-berlin.de

Example sheet 5

Due May 14, after the lecture

Problem 1 [to be submitted]

Show that, if $n > s \cdot r \cdot p$, then any sequence of n real numbers must contain either a strictly increasing subsequence of length greater than s, a strictly decreasing subsequence of length greater than r, or a constant subsequence of length greater than p.

Problem 2 []

(a) Prove that $1 + t < e^t$ for all $t \neq 0$.

(b) Let $k, n \in \mathbb{N}$. Prove $\binom{n}{k} < \left(\frac{ne}{k}\right)^k$.

Problem 3 []

(a) Write down the definition of $R(k, k, k)$, which is the generalization of the Ramsey number to three colors.

(b) Generalize the proof (from the lecture) of the lower bound for $R(k, k)$ to the three colors and show (for $k \geq 2$):

$$R(k, k, k) > \frac{k}{3e}\sqrt{3}^k.$$

Problem 4 []

Let $R_r(3)$ be the generalization of the Ramsey number $R(3, 3)$ to r many colors.

(a) Write down the definition of $R_r(3)$.

(b) Show that $R_r(3)$ is finite, by first proving the inequality $R_r(3) \leq r(R_{r-1}(3) - 1) + 2$.

(c) Prove the following upper bound:

$$R_r(3) \leq \lfloor e \cdot r! \rfloor + 1.$$

Problem 5 []

Construct for every $k \in \mathbb{N}$ an edge-coloring of the complete graph with $(k-1)^2$ vertices, which shows that $R(k, k) > (k-1)^2$.