email: person@math.fu-berlin.de
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Example sheet 6

Due May 21, after the lecture

Problem 1 [to be submitted]

Let a_n be the number of ways of paying the sum of n dollars using coins of values 1, 2, and/or bills of 5 dollars. Write down the generating function for the sequence $(a_0, a_1, \ldots)$ in closed form (with explanations, of course!).

Problem 2 []

(a) Find a recurrence relation for the number of sequences of 0s and 1s of length n that do not contain three consecutive 0s. What are the initial conditions?

(b) Solve the recursion $a_{n+3} = a_{n+2} + 9a_{n+1} - 9a_n$ for all n with the initial conditions $a_0 = 0$, $a_1 = 1$ and $a_2 = 2$.

Problem 3 []

Let F_n be the Fibonacci numbers defined in the lecture ($F_0 = 0$, $F_1 = 1$, and generally: $F_{n+2} = F_{n+1} + F_n$). Find out what the values of the following sums should be (by evaluating smaller sums) and prove the identities by induction:

(a) $F_1 + F_3 + \ldots + F_{2n-1}$.

(b) $F_0 + F_2 + \ldots + F_{2n}$.

Problem 4 []

(a) Find a recurrence relation for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable.

(b) What are the initial conditions for the recurrence relation in (a)?

(c) How many ways are there to lay out a path of fifteen tiles as described in (a)? How does the general formula look like?

Problem 5 []

Find the generating function for the sequence (a_n) given by

$$a_{n+2} = 3a_{n+1} + 4a_n$$

with $a_1 = 1$ and $a_2 = 3$. And from this find a formula for a_n.