

Example sheet 7

Due May 29, after the lecture

Problem 1 [to be submitted]
Find generating functions for the following sequences (express them in a closed form, without infinite series):

- (a) $0, 0, 0, 0, -6, 6, -6, 6, -6, \dots$
- (b) $1, 0, 1, 0, 1, 0, \dots$
- (c) $1, 2, 1, 4, 1, 8, \dots$
- (d) $1, 1, 0, 1, 1, 0, 1, 1, 0, \dots$

Problem 2 □
Find the generating functions for the sequences $(a_n)_{n \in \mathbb{N}_0}$, where

- (a) $a_n = n(n - 1)$,
- (b) $a_n = n^2$,
- (c) $a_n = n^3$.

Problem 3 □
Let a_n be the number of sequences of 0s, 1s and 2s of length n in which a 0 can only be followed by a 1. Find the recurrence relation which the sequence (a_n) satisfies, and the generating function for the sequence.

Problem 4 □
A sequence of numbers $a_0 := 1, a_1, \dots$ satisfies the conditions

$$a_{n+1} = 2a_n + n \quad (n \geq 0).$$

Find the generating function of this sequence (closed form) and a formula for the general term a_n .

Problem 5 □
We want to break a stick of length n into n pieces of unit length. What is the number of ways of doing so if

- (a) at each step, we break one of the pieces with a length greater than 1 into two,
- (b) at each step, we break all pieces with a length greater than 1 into two.