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Example sheet 8

Due June 4, after the lecture

A few definitions from the end of the lecture:

- The diameter of a graph G, $\mathrm{diam}(G)$, is the maximum of $d_G(x,y)$ over all $x, y \in V(G)$. Here, $d_G(x,y)$ denotes the length of the shortest path between x and y, if no such path exists then set $d_G(x,y) = \infty$.
- The complement of $G = (V,E)$ is denoted by $\bar{G}$ and defined as $(V, \binom{V}{2} \setminus E)$.

Problem 1 [to be submitted]

Let $d \in \mathbb{N}$ and $V := \{0,1\}^d$; thus, V is the set of all 0-1-sequences of length d. The graph on V in which two such sequences form an edge if and only if they differ in exactly one position is called the d-dimensional cube Q_d. Determine the average degree, number of edges, diameter, girth and circumference of this graph. Further, how can you describe a graph on the vertex set $\mathcal{P}([d])$ (power set of $[d]$), which is isomorphic to Q_d?

Problem 2 []

Let G be a graph containing a cycle C, and assume that G contains a path of length at least k between (some) two vertices of C. Show that G contains a cycle of length at least $\sqrt{k}$.

Problem 3 []

(a) Show that if G is a graph with n vertices and more than $\binom{n-1}{2}$ edges then G is connected. For $n > 1$, find a disconnected graph with n vertices and $\binom{n-1}{2}$ edges.

(b) Let G be a graph and $v(G) > 1$. Show that either G or $\bar{G}$ is connected.

Problem 4 []

Let S be a set of n points in the plane, the distance between any two of which is at least one. Show that there are at most $3n$ pairs of points of S at distance exactly one.

Problem 5 []

A graph is *self-complementary* if it is isomorphic to its complement. Show that:

(a) each of the graphs P_3 and C_5 is self-complementary,

(b) every self-complementary graph is connected,

(c) if G is self-complementary, then $v(G) \equiv 0$ or $1 (\mathrm{mod}\ 4)$,

(d) every self-complementary graph on $4k+1$ vertices has a vertex of degree $2k$.