Example sheet 9

Due June 11, after the lecture

Problem 1   [to be submitted]
Let T be a tree and G be a graph with at least $(|T| - 1)|G|$ edges. Show that G has a subgraph isomorphic to T.

Problem 2   []
Show that every connected graph G contains a path of length at least $\min\{2\delta(G), |G| - 1\}$.

Problem 3   []
Show that every automorphism of a tree fixes a vertex or an edge.

Problem 4   []
Prove or disprove that every connected graph contains a walk that traverses each of its edges exactly once in each direction.

Problem 5   []
Prove the equivalent characterizations of the tree from the lecture:

1. T is a tree.

2. Any two vertices of T are linked by a unique path in T.

3. T is minimally connected.

4. T is maximally acyclic.

5. T is connected and $|T| = \parallel T \parallel + 1$.