DISCRETE MATHEMATICS 1 SOMMERSEMESTER 2012

email: person@math.fu-berlin.de 2 JULY 2012

Example sheet 13

Due Juli 10 (no submission!)

Problem 1 []

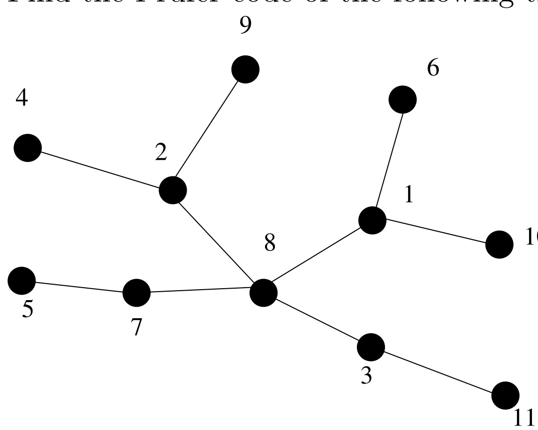
Show that the following statements about the line graph $L(G)$ of a graph G:

(a) G is eulerian $\Longrightarrow L(G)$ is eulerian.

(b) G is hamiltonian $\Longrightarrow L(G)$ is hamiltonian.

Problem 2 []

(a) Find the Prüfer code of the following tree:



(b) Find a tree with the Prüfer code $(3, 1, 4, 1, 5, 9, 2, 6, 5)$.

Problem 3 []

How many spanning trees does the graph $K_n - e$ possess? ($K_n - e$ is the complete graph minus an arbitrary edge.)

Problem 4 []

Show that the number of labeled graphs with the vertex set $[n]$, whose degrees are all even is $2^{\binom{n-1}{2}}$.
Hint: find an appropriate bijection.

Problem 5 []

We have shown earlier that the Ramsey number $R_r(3) \leq \lfloor e \cdot r! \rfloor + 1$. Give the following exponential lower bound: $R_r(3) \geq 2^r$ (i.e. color the edges of K_{2^r} with r colors without a monochromatic K_3).
Warning: this is not an exercise about *proper* edge colorings!

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Example sheet 14

Due Juli 10 (no submission!)

Problem 1 []

(a) Let G be a connected graph, which contains exactly $2k$ vertices of odd degree. Show that one can partition $E(G)$ in exactly k trails.

(b) What can be said about graphs with exactly $2k+1$ vertices of odd degree?

Problem 2 []

Let $d_1, \ldots, d_n$ be natural numbers and $n \geq 2$. Show that there is a tree with vertex degrees $d_1, \ldots, d_n$ if and only if $\sum_{i=1}^{n} d_i = 2n-2$.

Problem 3 []

Let T and T' be two spanning trees of a connected graph G. Show that for every $e \in E(T) \setminus E(T')$ there is an edge $e' \in E(T') \setminus E(T)$ such that the graphs $T' + e - e'$ and $T - e + e'$ are again spanning trees in G.

Problem 4 []

Show that if the edge weights of a connected graph are pairwise distinct, then there is a unique minimum spanning tree in G.

Problem 5 []

The adjacency matrix $A_G = (a_{ij})_{n\times n}$ of a graph G on the vertex set $V = \{v_1, \ldots, v_n\}$ is defined as follows: $a_{ij} = 1$ if ij is an edge of G and $= 0$ otherwise. Show that the matrix $A_G^k = (a'_{ij})_{n\times n}$ displays for all $i, j \leq n$, the number a'_{ij} of walks of length k from v_i to v_j in G.