Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

Summer Semester 2014
24 Junel 2014

Lecturer: Tibor Szabó
Tutor: Olaf Parczyk

Mock Final

Show all your work and state precisely the theorems you are using from the lecture. Ideally, try to solve the sheet within a time limit of 3 hours, without using any books, notes, etc ... (but of course this is not mandatory if you feel it would not yet make sense this way). It will be graded like the Final Exam, but the points do not count towards your exercise credit.

Submit by the 4th of July, 2PM in the box of Olaf Parczyk.

Exercise 1 [10 points]

(a) Define the Stirling number $S_{n,k}$ of the second kind.

(b) Show that $S_{n+1,k+1} = \sum_{i=0}^{n} \binom{n}{i} S_{i,k}$ for every $n, k \geq 0$.

Exercise 2 [10 points]

Show that the number

$$\frac{1}{2}\left((1+\sqrt{2})^n + (1-\sqrt{2})^n\right)$$

is an integer for every $n \geq 0$.

Exercise 3 [10 points]

(a) Define the generalization $R_r(3) := R(\underbrace{3, 3, \ldots, 3}_{r})$ of the Ramsey number $R(3,3)$ for r colors.

(b) Show that $R_r(3)$ is finite by proving the inequality $R_r(3) \leq r(R_{r-1}(3) - 1) + 2$.

Exercise 4 [10 points]

- Define what is a bipartite graph. Define what is an Eulerian graph.
- Prove or disprove: Every Eulerian bipartite graph has an even number of edges

Exercise 5 [10 points]

Prove Dirac's theorem from the lecture: If $\delta(G) \geq \frac{n}{2}$ for a graph G on $n \geq 3$ vertices, then G is Hamiltonian.

Exercise 6 [10 points]

Show that a sequence $(d_1, \ldots, d_n) \in \mathbb{N}_0^n$ is the degree sequence of a tree if and only if $\sum_{i=1}^{n} d_i = 2n - 2$.