Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

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Exercise sheet -1

To be solved and discussed during the first week exercise session

Exercise 1 []

Let $D(n)$ be the number of fixed point free permutations of the set $[n]$. What is wrong with the following inductive proof that $D(n) = (n-1)!$ for all $n \geq 2$?
`For` $n = 2$`, the formula holds, so assume` $n \geq 3$`. Let` π `be a permutation of` $[n-1]$ `with no fixed point. We want to extend it to a permutation` π' `of` $[n]$ `with no fixed point. We choose a number` $i \in [n-1]$`, and we define` $\pi'(n) = \pi(i)$`,` $\pi'(i) = n$ `and` $\pi'(j) = \pi(j)$ `for every` $j \neq i, n$`. This defines a permutation of` $[n]$ `and it is easy to check that it has no fixed point. For each of the` $D(n-1) = (n-2)!$ `possible choices of` π`, the index` i `can be chosen in` $n-1$ `ways. Therefore,` $D(n) = D(n-1) \cdot (n-1) = (n-2)! \cdot (n-1) = (n-1)!$.

Exercise 2 []

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and groom are among these 10 people, if

(i) the bride must be in the picture?

(ii) both the bride and groom must be in the picture?

(iii) exactly one of the bride and the groom is in the picture?

Exercise 3 []

How many positive integers less than 1,000,000 have exactly one digit equal to 9 and have a sum of digits equal to 13?

Exercise 4 []

Read carefully the following induction proof:
Assertion If we have n lines in the plane, no two of which are parallel, then they all go through one point.
Proof The assertion is true for one lion (and also for 2, since we have assumed that no two lines are parallel). Suppose it is true for any set of n lines, using this induction hypothesis.
So consider a set $S = \{a, b, c, d, \ldots\}$ of n lines in the plane, no two of which are parallel. Delete the line c; then we are left with a set S' of $n-1$ lines, and obviously,

no two of these are parallel. So we can apply the induction hypothesis and conclude that there is a point$P$ such that all the lines in S' go through P. In particular, a and b go through P, and so P must be the point of intersection of a and b.
Now put c back and delete d, to get a set S'' of $n-1$ lines. Just as above, we can use the induction hypothesis to conclude, that these lines go through the same point P'; but just as above, P' must be the point of intersection of a and b. Thus $P' = P$. But then we see that c goes through P. The other lines also go through P (by the choice of P), ans so all the n lines go through P.
But the assertion we proved is clearly wrong; where is the error?

Exercise 5 []

There are 33 very intelligent and very hungry lions on an island. A piece of meat is dropped on the island, injected with an extremely powerful sleeping drug. All the lions know about the drug and the other lions. The stuff in the meat is so strong that its effect does not go away: even after a lion ate the meat and fell asleep, and some other lion eats him, the drug effects the other lion as well and he also falls asleep, having the drug (still effective!) in his belly. If another lion eats him, etc... Each lion faces a choice: either he wants to eat the meat to ease his hunger, or he does not want to do anything (just stare hungrily) because he does not want to risk that while falling asleep some other lion would eat him. (We assume that lions dont share: the meat or a sleeping lion can be eaten only by one other lion.) What will the lions do? The order of preference of each lion is, of course: 1. Wants to stay alive. 2. Wants to eat. Will the lions try to run for the meat, so they can be the one who gets to eat it, or they choose to stay hungry?