Exercise sheet 0

To be solved and discussed during the second week exercise sessions

Exercise 1

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ where x_i, $i = 1, 2, 3, 4, 5, 6$ is a nonnegative integer such that

- $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?

- $x_1 < 8$ and $x_2 > 8$

Exercise 2

What is the probability that a 5-card poker hand (drawn randomly from a 52-card deck) contains

- a flush (five cards in the same suit)?

- a straight (five cards of sequential kind)?

- cards of five different kinds and it does not contain a flush or a straight?

Exercise 3

How many ways are there to travel in the 3-dimensional Euclidean space from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by taking steps one unit in the positive x direction or one unit in the positive y direction or one unit in the positive z direction? (Moving in the nagtive directions is prohibited.)

Exercise 4

What is the coefficient of $x^6 y^4 z^{11}$ in the expansion of $(3x + 4y - 5z)^{21}$?

Exercise 5

Give an algebraic and a combinatorial proof that

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}.$$