

LECTURER: TIBOR SZABÓ  
TUTOR: OLAF PARCZYK

## *Exercise sheet 1*

Submit by 25 April, 2PM in the box of Olaf Parczyk

**Exercise 1** [10 points]

How many ways can we partition a set  $A$  into  $k$  subsets  $A_1, \dots, A_k$ ? (The sets are allowed to be empty and the index of the sets does matter.)

**Exercise 2** [10 points]

Let  $A \in \binom{[100]}{55}$ . Show that  $A$  contains two numbers whose difference is 9.  
Is this also true if  $|A| = 54$ ?

**Exercise 3** [10 points]

What is the largest binomial coefficient  $\binom{n}{k}$  for a fixed  $n$ ?

**Exercise 4** [10 points]

Give a combinatorial proof that

$$\sum_{k=1}^n k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.$$