FREIE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK
DISCRETE MATHEMATICS 1

SUMMER SEMESTER 2014
17 JUNE 2014

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Exercise sheet 10

Submit only **four(!!!)** exercises by 27th of June, 2PM in the box of Olaf Parczyk

Exercise 1 [10 points]
Prove that the number of simple graphs on the vertex set $[n]$ in which all degrees are even is $2^{\binom{n-1}{2}}$. (*Hint:* Try to find a bijective proof.)

Exercise 2 [10 points]
An edge e of a graph G is called a *cut-edge* if its removal increases the number of connected components. Prove that a graph with all vertices having even degree does not contain a cut-edge. For each $k \in \mathbb{N}$ construct a simple $(2k+1)$-regular graph having a cut-edge.

Exercise 3 [10 points]
Let P and Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex.

Exercise 4 [10 points]
Let G be a connected graph not having P_4 and C_3 as an induced subgraph. Prove that G is a complete bipartite graph.

Exercise 5 [10 points]
Each game of bridge involves two teams of two partners each. Consider a club, where four players cannot play a game if any two of them have previously been partners that night. Suppose that fifteen members arrive, but one decides to study graph theory instead. The other fourteen people play until each has played four times. Next the rules allow them to play six more games. Prove that if the graph theorist now agrees to play, then at least one more game can be played.

Exercise 6 [10 points]
Determine the minimum number of edges in an n-vertex connected graph in which every edge is contained in a triangle.