FREIE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK
DISCRETE MATHEMATICS 1

SUMMER SEMESTER 2014
1 JULY 2014

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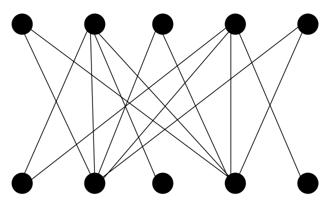
Exercise sheet 11 — BONUS

Submit only **four(!!!)** exercises by 11th of July, 2PM in the box of Olaf Parczyk

Exercise 1 [10 points]
Determine the size of the largest matching in the graph below.



Exercise 2 [10 points]
Show that a tree contains at most one perfect matching. Give a tree on an even number of vertices which has no perfect matching.

Exercise 3 [10 points]
How many spanning trees does $K_n - e$ have? ($K_n - e$ denotes the complete graph minus one edge.)

Exercise 4 [10 points]
Prove that a bipartite graph of order n contains a perfect matching if, and only if $\alpha(G) = n/2$. (Here $\alpha(G)$ denotes the maximum cardinality of an independent set of vertices in G).

Exercise 5 [10 points]
Determine the characteristic polynomial of the sequence p_n, where p_n is the number of perfect matchings in the $3\times(2n)$ grid graph G, defined as follows: $V(G) := [3]\times[2n]$ and

$$E(G) := \{\{(i,j),(i,j+1)\} : i \in [3], j \in [2n-1]\} \cup \{\{(i,j),(i+1,j)\} : i \in [2], j \in [2n]\}.$$

Give a closed formula for p_n.

Exercise 6 [10 points]
Generalizing Tic-Tac-Toe. A *positional game* consists of a set $X = \{x_1, \dots, x_n\}$, the *board*, and designated subsets $W_1, \dots, W_m \subseteq X$ of the board, the *winning sets*.

(Traditional 3×3 Tic-Tac-Toe has a board with nine elements and eight winning sets: the horizontal, vertical and diagonal lines.) Two players alternately choose elements of X; a player wins by choosing all elements of a winning set first.

Suppose that each winning set has size at least 10 and each element of the board appears in at most 5 winning sets. Prove that Second Player can force at least a draw.(*Hint:* Show that Second Player can find a family of disjoint pairs of elements of the board such that each winning set contains at least one of these pairs and explain how he could use such a pairing to draw the game. (Such a strategy is called a *pairing strategy.*)