Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

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Exercise sheet 12

Solve in preparation for final, do not submit

Exercise 1 []
For any $0 < k \leq l \leq m$, give a construction of a graph G with $\kappa(G) = k$, $\kappa'(G) = l$, $\delta(G) = m$. (Don't forget to argue properly for the correctness of your construction!)

Exercise 2 []

(a) Let G' be a supergraph of a k-connected graph G obtained by adding one vertex to $V(G)$ with at least k neighbors. Show that G' is k-connected as well.

(b) Show that if G is 2-connected then every pair of edges lie on a common cycle.

Exercise 3 []
Prove that a simple graph G is 2-connected if and only if for every ordered triple (x, y, z) of distinct vertices, G has an x, z-path through y.

Exercise 4 []
Show that for any graph G, we have $\chi(G) + \chi(\bar{G}) \leq v(G) + 1$.

Exercise 5 []
Prove that if G is k-color-critical, then the Mycielski graph $M(G)$ of G is $(k+1)$-color-critical.

Exercise 6 []
Prove that minimum number of edges a k-chromatic graph can have is $\binom{k}{2}$. Use this to prove that if G is contained in the union of m copies of K_m then $\chi(G) \leq m^{3/2}$.

Remark: The Erdős-Faber-Lovász Conjecture (1972) states that if the m copies of K_m are *pairwise edge-disjoint* then $\chi(G) = m$. Paul Erdős offered \$ 500 for a proof. (after Erdős' death the pledges he made for the resolution of various mathematical problems are still honored by Ron Graham.)