Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

Summer Semester 2014
25 July 2014

Lecturer: Tibor Szabó
Tutor: Olaf Parczyk

Exercise sheet 13

Solve in preparation for the second final, do not submit

Definition A planar graph G is *outerplanar* if there is an embedding of it in the plane such that all vertices are on the boundary of the outer face.

Exercise 1 []

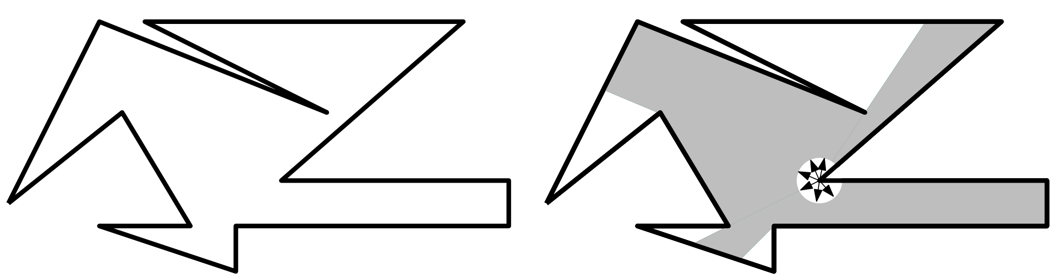
Use Kuratowski's Theorem to show that a graph is outerplanar if and only if it does not contain a subdivision of K_4 or $K_{2,3}$.

Exercise 2 []

Prove, without using the Four Color Theorem, that every outerplanar graph is 3-colorable.

Exercise 3 []

Apply Problem 2 to prove the Art Gallery Theorem: If an art gallery is laid out as a simple polygon with n sides, then it is possible to place $\lfloor n/3 \rfloor$ guards such that every point of the interior is visible by some guard.
Construct a polygon that does require $\lfloor n/3 \rfloor$ guards.



An art gallery and what a guard sees from a corner

–

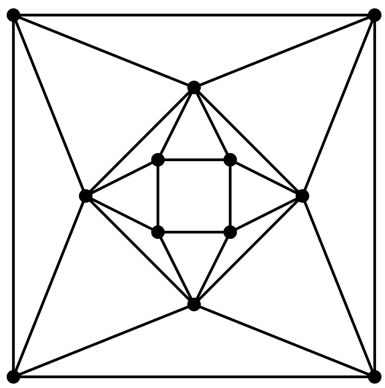
Exercise 4 []

Prove that every simple planar graph with at least four vertices has at least four

vertices with degree less than 6.
For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than 6.

Exercise 5 []

Define a sequence of plane graphs as follows. Let $G_1 = C_4$. For $n > 1$ obtain G_n from G_{n-1} by adding a new 4-cycle surrounding G_{n-1}, making each vertex of the new cycle also adjacent to two consecutive vertices of the previous outside face. The graph G_3 is shown below.
Prove that if n is even, then every proper 4-coloring of G_n uses each color on exactly n vertices.



Exercise 6 []

(a) Give a drawing of K_6 in the real projective plane without any crossing. (Think of the projective plane as a closed disc where opposite points of the boundary circle are identified.)

(b) Give a drawing of K_7 on the torus without any crossing. (Think of the torus as the unit square $[0,1]^2$, where each boundary point $(0,y)$ is identified with $(1,y)$ and point $(x,0)$ is identified with $(x,1)$.)