Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

Lecturer: Tibor Szabó
Tutor: Olaf Parczyk

Summer Semester 2014
15 April 2014

Exercise sheet 2

Submit by 2nd of May, 2PM in the box of Olaf Parczyk

Exercise 1 [10 points]
How many k-subset of $[n]$ are there containing no two consecutive numbers?

Exercise 2 [10 points]
Let $f(n,k)$ be the number of sequences $(a_1,\dots,a_n)$ of positive integers such that the largest entry is k and an entry i appears at least once before the first occurrence of $i+1$ for every $i, 1 \le i \le k-1$. Show that $f(n,k) = S_{n,k}$ using a bijection.
(Be very explicit and formal in defining the two sets and the function between them, as well as in your proof that the function you defined is indeed a bijection.)

Exercise 3 [10 points]
Show that $\sum_{k=0}^{n} S_{n+1,k+1} x^{\underline{k}} = (x+1)^n$, and use the polynomial method to prove $S_{n+1,k+1} = \sum_{i=0}^{n} \binom{n}{i} S_{i,k}$. Verify this last equality also by a combinatorial argument, and deduce again the Bell number recurrence:

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

Exercise 4 [10 points]
Show the following surprising formula for the Bell numbers ($n \in \mathbb{N}$):

$$B_n = \frac{1}{e} \sum_{i=0}^{\infty} \frac{i^n}{i!}$$