Freie Universität Berlin
Institut für Mathematik
Discrete Mathematics 1

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Exercise sheet 3

Submit by 9th of May, 2PM in the box of Olaf Parczyk

Exercise 1 [10 points]

Let $c_n^{(r)}$ be the number of those permutations of $[n]$ which only have cycles of length at most r in their cyclic representation. Prove the recurrence $c_{n+1}^{(r)} = \sum_{k=n-r+1}^{n} n^{\underline{n-k}} c_k^{(r)}$.

Exercise 2 [10 points]

What is the expected number of fixed points when all permutations of $[n]$ are equally likely? (*Hint:* Use the linearity of expectation.)

Exercise 3 [10 points]

Prove, using a bijection, that the number of partitions of n into distinct terms is the same as the number of partitions of n into odd terms. (*Hint:* Consider the unique representation of numbers as the product of an odd number and a power of 2.)

Exercise 4 [10 points]

Show that the number of self-conjugate partitions, i.e., those with $\lambda^* = \lambda$, equals the number of partitions of n with all summands odd and distinct.