FREIE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK
DISCRETE MATHEMATICS 1

SUMMER SEMESTER 2014
13 MAY 2014

LECTURER: TIBOR SZABÓ
TUTOR: OLAF PARCZYK

Exercise sheet 5

Submit by 23rd of May, 2PM, in the box of Olaf Parczyk

Exercise 1 [10 points]
Let a_n be the number of ordered triples (i, j, k) of integers such that $i \geq 0, j \geq 2, k \geq 3$, and $i + 3j + 3k = n$. Find the generating function of the sequence $(a_0, a_1, \ldots)$ and use it to calculate a formula for a_n.

Exercise 2 [10 points]
Prove that the number of triangulations of a convex n-gon (with labeled vertices) is the Catalan number b_{n-2}. (A triangulation is a partition of the n-gon into triangles using $n-3$ non-intersecting diagonals (a diagonal is segment connecting two vertices.))

Exercise 3 [10 points]
How many rooted binary trees are there on n vertices where each vertex has either zero or two children? (Recall: formally a rooted binary tree on n vertices is an ordered pair of rooted binary trees B_ℓ and B_r, such that there is some $i, 0 \leq i \leq n-1$, for which B_ℓ is a rooted binary tree on i vertices and B_r is a rooted binary tree on $n-1-i$ vertices. The *children* of a vertex are the roots of its left and the right subtree.)

Exercise 4 [10 points]
Order the following expressions according to growth rate and express this ordering using asymptotic notation: $n \ln n, (\ln \ln n)^{\ln n}, (\ln n)^{\ln \ln n}, n \cdot e^{\sqrt{n}}, (\ln n)^{\ln n}, n \cdot 2^{\ln \ln n}, n^{1+1/(\ln \ln n)}, n^{1+1/\ln n}, n^2$. (This is an exceptional exercise: you need to argue only for yourself, but to write, the final answer is sufficient!)