FREIE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK
DISCRETE MATHEMATICS 1

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Exercise sheet 7

Submit only **four(!!!)** exercises by 6th of June, 2PM in the box of Olaf Parczyk

Exercise 1 [10 points]

In the lecture we defined the incidence algebra $\mathbb{A}(P)$ for any locally finite poset P and proved that the convolution product $f \star g$ is associative. Show that

(a) the Kronecker delta, defined by

$$\delta(a,b) := \begin{cases} 1 & \text{if } a = b \\ 0 & \text{otherwise} \end{cases}$$

is the unique identity element (both from left and right).

(b) Let $f \in \mathbb{A}(P)$ be such that $f(x,x) = 1$ for every $x \in P$. Define the function $f_L^{-1} : P \times P \to \mathbb{C}$ (inductively, on the size of the interval $[a,b] \subseteq P$) as follows. Let $f_L^{-1}(a,a) := \frac{1}{f(a,a)}$ for every $a \in P$. Suppose $f_L^{-1}(a,z)$ is already defined for every $z \in [a,b] \setminus \{b\}$ and let

$$f_L^{-1}(a,b) := \frac{1}{f(b,b)} \left(- \sum_{\substack{z \in P \\ a \leq z < b}} f^{-1}(a,z) f(z,b) \right)$$

Show that f_L^{-1} is the unique left-inverse of f.

(c) Define analogously a function f_R^{-1} and show that it is the unique right-inverse of f. Prove that $f_L^{-1} = f_R^{-1}$.

Exercise 2 [10 points]

(a) Choose a locally finite poset P and functions $f, g : P \to \mathbb{C}$ to show that the inclusion-exclusion formula is a special case of the Möbius Inversion over the incidence algebra $\mathbb{A}(P)$.

(b) Prove that the number theoretic Möbius inversion is the special case of the one over incidence algebras.

Exercise 3 [10 points]

Prove that among the 33 guests at a party there are two who know the same number of the other guests.

Exercise 4 [10 points]

The reigning champion of an endurance blitz chess championship must start playing at least one match every hour (in the time interval $[i, i+1)$ for $i \in \mathbb{N}_0$). The current champion has done that for 75 consecutive hours. Show that if he did not play more than 125 matches, then there was a period of consecutive hours during which he started exactly 24 matches.

Bonus challenge: For how many integers $m \in [75]$ can you show the above statement with 24 exchanged to m? The solution with the most successful m wins 5 bonus points!)

Exercise 5 [10 points]

Show that, if $n > s \cdot r \cdot p$, then any sequence of n real numbers must contain either a strictly increasing subsequence of length greater than s, a strictly decreasing subsequence of length greater than r, or a constant subsequence of length greater than p.