

LECTURER: TIBOR SZABÓ
TUTOR: OLAF PARCZYK

Exercise sheet 8

Exercise 1

[10 Punkte]

Determine which pairs of graphs from below are isomorphic. Check as few pairs as possible.



Exercise 2

[10 points]

- (a) Show that $R(3, 4) \geq 9$. Start with writing down precisely in a sentence what you will need to do.
- (b) Show that $R(3, 4) \leq 10$.
- (c) Improve (b) further and show that $R(3, 4) \leq 9$.

Exercise 3

[10 points]

- (a) Define the Ramsey number $R(k, \ell, m)$, the generalization of the Ramsey number to three colors.
- (b) Show that for $k \geq 2$ we have

$$R(k, k, k) > \frac{k}{3e} \sqrt{3}^k.$$

Start with writing down precisely in a sentence what you need to show and at the end conclude why/how you are done doing that.

Exercise 4

[10 points]

- (a) Define the generalization $R_r(3) := R(\underbrace{3, 3, \dots, 3}_r)$ of the Ramsey number $R(3, 3)$ for r colors.

(b) Show that $R_r(3)$ is finite by proving the inequality $R_r(3) \leq r(R_{r-1}(3) - 1) + 2$. Start with expressing precisely in a sentence what you need to show.

(c) Prove the following upper bound:

$$R_r(3) \leq \lfloor e \cdot r! \rfloor + 1.$$