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TUTOR: OLAF PARCZYK

## *Exercise sheet 9*

Submit by 20th of June, 2PM in the box of Olaf Parczyk

**Exercise 1** [10 points]

Construct a bipartite graph with no nontrivial automorphism.

**For 5 Bonus points** Determine the smallest number  $k$  such that a graph on  $k$  vertices with no nontrivial automorphism exist.

**Exercise 2** [10 points]

Show that the Petersen graph contains no 7-cycle.

**Exercise 3** [10 points]

A graph is *self-complementary* if it is isomorphic to its complement. Show that there exists a self-complementary graph on  $n$  vertices if and only if  $n \equiv 0$  or  $1 \pmod{4}$ .

**Exercise 4** [10 points]

A graph  $G$  is called *vertex-transitive* if for any two vertices  $x, y \in V(G)$ , there is an automorphism  $\phi \in \text{Aut}(G)$  such that  $\phi(x) = y$ . A graph  $G$  is called *edge-transitive* if, for any two edges  $uv, xy \in E(G)$ , there is an automorphism  $\phi \in \text{Aut}(G)$  such that  $\phi(u)\phi(v) = xy$ .

- (a) Find a graph which is vertex-transitive but not edge-transitive.
- (b) Show that any graph without isolated vertices which is edge-transitive but not vertex-transitive is bipartite.