Discrete Mathematics 1 | Summer Term 2015
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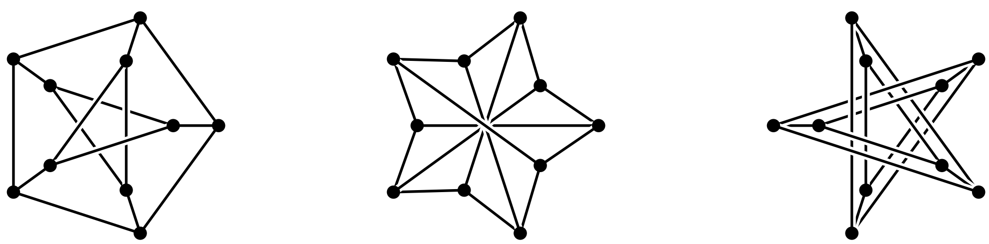
Mock Exam

Due 10AM, **Friday, 26 June 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

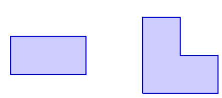
Show all your work and state precisely the theorems you are using from the lecture. Ideally, try to solve the sheet within a time limit of 90 minutes, without using any books, notes, etc ... (but of course this is not mandatory if you feel it would not yet make sense this way). It will be graded like the Final Exam, but the points do **not** count towards your exercise credit.

Problem 1 [10 points]
Prove the following statement for each graph G: Either G or its complement $\overline{G}$ is connected.

Problem 2 []
Prove which of the following graphs are isomorphic to each other and which are not.



Problem 3 [10 points]
Write down a recurrence relation for the sequence c_n, where c_n represents the number of ways one can cover a $n \times 2$-rectangle completely without overlap with the following two types of tiles:



The tiles my be rotated by integral multiples of 90 degree. It is not necessary to *solve* the recurrence.

Problem 4 [10 points]

(a) Define the Ramsey number $R(k, l)$.

(b) Prove that $R(k, k) > \sqrt{2}^k$ for every large enough k. (The statement is true for every k, but here it is enough if you show it for, say, $k \geq 10$).

Problem 5 [10 points]

(a) Define the Stirling numbers $S_{n,k}$ of the second kind.

(b) Prove that it holds that

$$S_{n,3} = \frac{3^n - 3 \cdot 2^n + 3}{6}$$

Problem 6 [10 points]

For $n \in \mathbb{N}$, let i_n denote the number of permutations $f \in S_n$ having the property $f(f(x)) = x$ for all $x \in [n]$. Define $i_0 := 1$. Prove the recurrence

$$i_n = i_{n-1} + (n-1)i_{n-2}$$

and find the exponential generating function of the sequence.