In Class Sheet 0

Problem 1   [In class]
How many anagrams of the word ZANDERBEZWINGZWANG are there?

Problem 2   [In class]
A country uses number plates with five digits. To reduce the number of digits the administration wants to use letters instead of numbers. How many digits have to be used at least to label at least the same number of cars with number plates?

Problem 3   [In class]
We want to to send postcards to n friends and can choose in a shop among k scenes on the postcards. Give a closed formula for the number of possibilities

1. if every friend is supposed to receive exactly one postcard?

2. ..., if every friend is supposed to get exactly two different postcards?

3. ..., if every friend should receive up to k postcards?

Problem 4   [In class]
How many non-negative integer solutions $(x_1, \ldots, x_5)$ does the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 133$ have? How does the answer change if we require that $x_4 \geq 5$ and $x_1 < 8$?

Problem 5   [In class]
Prove in two different ways:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 6   [In class]
Prove in two different ways:

$$r \binom{n}{r} = n \binom{n-1}{r-1}$$