# Exercise sheet 1

**Due 2PM, Friday, 24 April 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

**Problem 1**                                                              [10 points]

- How many different ways are there to place five different kinds of symbols on $3 \times 3$ fields?
- How many different ways are there to place three symbols (for instance ✗, ○, and empty space) on $3 \times 3$ fields, so that one symbol is used $k$ times and the other $k + 1$ times?
- The game "Tic Tac Toe" is played by two players on $3 \times 3$ fields. The players occupy the fields in turns; their goal is to mark three fields in a row, in a column or in a diagonal. The game ends if all fields are marked or if one player has won. How many finished Tic-Tac-Toe games won by the first player are there?

|   | ✗ | ○ |
|---|---|---|
| ✗ | ○ | ○ |
| ✗ |   | ○ |

**Problem 2**                                                              [10 points]
How many ways are there to pick $k$ elements of $[n]$ in order, such that at the end there are no two consecutive elements picked?

**Problem 3**                                                              [10 points]
How many ways are there to travel in the 3-dimensional Euclidean space from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by taking steps one unit in the positive x direction or one unit in the positive y direction or one unit in the positive z direction? (Moving in the negtive directions is prohibited.)

**Problem 4**                                                              [10 points]
Prove in two different ways:

$$\sum_{k=r}^{n} \binom{n}{k}\binom{k}{r} = 2^{n-r}\binom{n}{n-r}$$

**Problem 5**                                                              [10 points]

(a) How many $n \times n$ matrices with entries from $\{0, 1, \ldots, q-1\}$ are there?

(b) Let $q$ be a prime number. How many non singular matrices are there over the field with $q$ elements? (In other words: How many matrices from question (a) have a determinant that is not divisable by $q$?)