Exercise sheet 10

Not be submitted, solutions to be discussed in the exercise sessions during the
week of June 29.

Problem 48

Let G be a graph containing a cycle C, and assume that G contains a path of length
at least k between (some) two vertices of C. Show that G contains a cycle of length
at least $\sqrt{k}$.

Problem 49

A graph is *self complementary* if it is isomorphic to its complement. Show that
a self complementary graph with n vertices exists if and only if $n \equiv 0 \bmod 4$ or
$n \equiv 1 \bmod 4$.

Problem 50

(a) Show that the Ramsey number $R(3,4) \leq 10$.
(b) Extend your proof and show $R(3,4) = 9$.

Problem 51

Show that a graph G if bipartite if and only if each subgraph H of G contains an
independent set of size at least $\frac{|V(H)|}{2}$.