Discrete Mathematics 1 — Summer Term 2015

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Exercise sheet 11

Due **2PM, Friday, 3 July 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

Problem 52 [10 points]
Show by induction that each connected component of a graph G is Eulerian if and only if the edges can be partitioned into disjoint sets, each of which induces a simple cycle in G.

Problem 53 [10 points]
Show that if for a graph G with n nodes $|E(G)| \geq \binom{n-1}{2} + 1$ holds, then G is connected.
Argue also that the statement is *best possible.* (In particular, formulate precisely your meaning of *best possible.*)

Problem 54 [10 points]
Prove that the number of simple graphs on the vertex set $[n]$ with all vertices having even degree is $2^{\binom{n-1}{2}}$.

Problem 55 [10 points]
Each game of bridge involves two teams of two partners each. Consider a club, where four players are not allowed to play a game if any two of them have previously been partners that night. Suppose that fifteen members arrive, but one decides to study graph theory instead. The other fourteen people play until each has played four times. After this, the rules allow them to play six more games.
Prove that if the graph theorist now agrees to play, then at least one more game can be played.

Problem 56 [10 points]

(a) Show that for any graph G with n nodes there is a subgraph $H \subseteq G$ such that

$$\delta(H) \geq \frac{|E(G)|}{n}.$$

(b) Let T be a tree on t vertices. Show that every graph G of n vertices with at least $(t-1)n$ edges has a subgraph isomorphic to T.

(c) Construct a graph on n vertices with at least $\frac{t-1}{2n}$ edges which does not contain *any* tree on t vertices.