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Exercise sheet 2

Due **2PM, Friday, 1 May 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

Problem 6 [10 points]
A pizza shop offers 7 toppings.
(a) How many possibilities are there to order 4 pizzas for a party?
(b) At the end of the day there is exactly one portion of each of the 7 toppings left. How many possibilities are there to fulfill the order?
(c) How does the problem change if the pizzas from question (b) are not for a party, but for four persons who come to the pizza shop?

Problem 7 [10 points]
The Bell number B_n is the number of all set partitions of a n-set, so $B_n = \sum_{k=0}^{n} S_{n,k}$, where $S_{n,k}$ are the Stirling number of the second kind and $B_0 := 1$. Show that

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$$

Hint: It could be useful to show that the following holds for Stirling numbers of the second kind:

$$S_{n+1,k+1} = \sum_{j=k}^{n} \binom{n}{j} S_{j,k}$$

Problem 8 [10 points]
Prove that it holds for Stirling numbers of the second kind that

$$S_{n,3} = \frac{3^n - 3 \cdot 2^n + 3}{6}.$$

Problem 9 [10 points]
Show that

$$\sum_{k=0}^{n-m} \binom{s+k}{k} \binom{n-k}{m} = \binom{s+n+1}{s+m+1}$$

holds for integral k, m, n, s.

Problem 10 [10 points]
In exercise sheet -1 you learned the following game:

> Given is a set of $m = 2^k$, $k \in \mathbb{N}$, lights which are sitting around a round disc that can be turned into any one of m positions. Each light has a switch that can turn the light on and off, independently of the others. A game between Alice and Bob runs as follows. Bob switches some of the lights on. In one round of the play, first Alice chooses a set of positions. Then Bob turns the disc by an angle of $2\pi i/m$ ($i \in \mathbb{N}$) of his choice and flips the switches of the lights at the positions Alice originally had chosen.

Prove now that if Alice always chooses the positions of all the lights that are turned on, she will eventually win.