Discrete Mathematics 1 — Summer Term 2015

email: andreas.loos@math.fu-berlin.de — 26/28 May 2012

Exercise sheet 5

Due **2PM, Friday, 22 May 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

Problem 22 [10 points]
Let (a_n) and (b_n) be two sequences, where

$$b_n = \sum_{i=0}^{n} a_i.$$

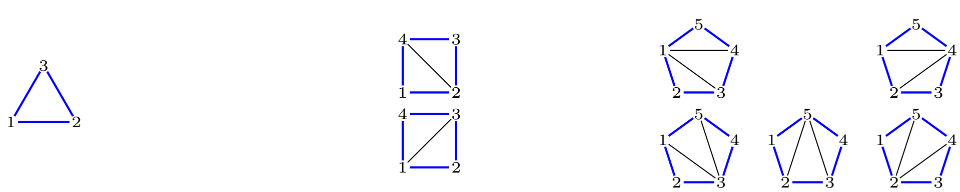
What is the relationship between their ordinary generating functions $A(x)$ and $B(x)$? What is the relationship between their exponential generating functions $\hat{A}(x)$ and $\hat{B}(x)$?

Problem 23 [10 points]
For $n \in \mathbb{N}$, let i_n denote the number of permutations $f \in S_n$ having the property $f(f(x)) = x$ for all $x \in [n]$. Define $i_0 := 1$. Prove the recurrence $i_n = i_{n-1} + (n-1)i_{n-2}$ and find the exponential generating function of the sequence.

Problem 24 [10 points]
Let t_n be the number of ways to arrange n books on two bookshelves such that each shelf contains at least one book. Use the generating function to derive a closed formula for t_n. Verify your answer by giving a direct combinatorial proof.

Problem 25 [10 points]
We select an odd number $k > 0$ of people from a group of n people to serve as a committee. From this committee we select an even number $\ell \geq 0$ to serve as a subcommitee. In how many ways can that be done?

Problem 26 [10 points]
Consider the triangulations of regular n-gons, here for $n = 3$, $n = 4$, and $n = 5$:



How many triangulations t_n are there for general n?