Discrete Mathematics 1 Summer Term 2015
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Exercise sheet 7

Due **2PM, Friday, 5 June 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

Problem 33 [10 points]
What is the number of integers smaller than one million that contain two consecutive digits which are the same?

Problem 34 [10 points]
Find a 216-element subset X of $[280]$ such that every 5-element subset of X contains two elements that are *not* relatively prime.
BONUS (10 points) Show that this is best possible, that is, every 217-element subset of $[280]$ has a 5-element subset with all pairs of elements being relatively prime.

Problem 35 [10 points]
For positive integers $1, 2, 3, \ldots, (n-1), n$ there are 11660 derangements where $1, 2, \ldots, 5$ appear in the first five positions. What is the value of n?

Problem 36 [10 points]
Find a closed formula for $\sum_{i=0}^{n} \binom{n}{i} D(i)$, where $D(i)$ is the number of derangements.

Problem 37 [10 points]
Let P be the product of a k-element chain and an m-element chain. What is the size of the largest chain and the largest antichain in P? (If $(R_1, \leq_1)$ and $(R_2, \leq_2)$ are two posets then their product is $(R_1 \times R_2, \leq)$, where $(x, y) \leq (x', y')$ if $x \leq_1 x'$ and $y \leq_2 y'$.)