email: andreas.loos@math.fu-berlin.de

Exercise sheet 9

Due **2PM, Friday, 19 June 2015**
in the mailbox of Andreas Loos (Villa Arnimallee 2) or via e-mail

Problem 43 [10 points]
Let A a set of n integer numbers. Then there is a subset $S \subseteq A$ such that the sum $\sum_{a \in S} a$ is divisible by n.

Problem 44 [10 points]
Let a_i the sequence of Fibonacci numbers and define $d_i := a_i (\mathrm{mod} 10)$. Prove that the d_i form a periodic sequence. Give the best bound on the period you can find without the help of a computer.

Problem 45 [10 points]
101 points are placed inside a 1×1 square. Show that some three of them form a triangle with area no more than 0.01.

Problem 46 [10 points]
Improve on the upper bound on the symmetric Ramsey number $R(k)$ by introducing the (two-variable) Ramsey number $R(k, \ell)$. Let $R(k, \ell)$ be the minimum integer N such that every 2-edge-coloring $c : E(K_N) \to \{\mathrm{Red}, \mathrm{Blue}\}$ of the complete graph K_N contains a red monochromatic clique on k vertices or a blue monochromatic clique on ℓ vertices.

(a) Show that $R(1, \ell) = 1 = R(k, 1)$ and $R(k, 2) = k = R(2, k)$.

(b) Show that for every $k, \ell \geq 2$, $R(k, \ell) \leq R(k-1, \ell) + R(k, \ell - 1)$.

(c) Using induction show that $R(k, \ell) \leq \binom{k+\ell-2}{k-1}$.

(d) Conclude that for the symmetric Ramsey number $R(k) = O\left(\frac{4^k}{\sqrt{k}}\right)$.

Problem 47 [10 points]

(a) Define the Ramsey numbers $R(k, k, k)$ for three colours.
(b) Prove that $R(k, k, k)$ is finite. (The very best upper bound gets bonus points!)
(c) Generalize the proof (from the lecture) of the lower bound for $R(k, k)$ to three colors and show (for $k \geq 2$) that

$$R(k, k, k) > \frac{k}{3e} \sqrt{3}^k.$$