Discrete Mathematics I So 2017–18
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Exercise Sheet 0

Do date: April 17th at 4:15 PM and April 18th at 10:15 AM.

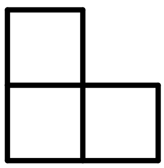
This sheet contains some practice problems to help get you warmed up for this semester's course. You are invited to work on them together and discuss your solutions during the exercise classes of the first week. Your solutions will not be graded.

Exercise 1 Say there is a closed path made up of 13 line segments in the Euclidean plane. Can there be a line, not containing any of the end points of the 13 line segments, such that this line intersects all 13 line segments?

Exercise 2 Find the flaws, if any exist, in the following proof:
Claim: For every positive integer n the following is true. If for two positive integers x and y we have $\max\{x, y\} = n$, then $x = y$.
Proof: We prove this by induction on n. For the base case of $n = 1$, if $\max\{x, y\} = 1$, then we must have $x = y = 1$ since $x \geq 1$ and $y \geq 1$. For the inductive step, suppose the statement is true for some $n \geq 1$. Now let x, y be positive integers such that $\max\{x, y\} = n + 1$. Then we have $\max\{x - 1, y - 1\} = n$, and so by the inductive hypothesis we have $x - 1 = y - 1$, which implies that $x = y$. $\square$

Exercise 3 From an 8×8 grid of unit squares, two diagonally opposite squares are removed. Prove that the remaining squares cannot be covered by 1×2 dominoes.

Exercise 4 Prove that for any $n \geq 1$, if any square in a $(2^n \times 2^n)$- grid is removed, then it can be perfectly tiled by L-shaped tiles.



Exercise 5 Into how many parts do n straight lines divide the Euclidean plane if no two of them are parallel and no three meet at the same point?

Exercise 6 Ali and Barb are on a game show. Each of them is told a secret positive integer by the host. They are both also told that the two numbers are consecutive, but neither person knows the other person's number. The point of the game is to guess the other person's number correctly.

Both of them are sitting in a room together where a clock rings after every 15 seconds. They are not allowed to communicate with each other, and of course every other form of cheating is disallowed as well. Moreover, they did not know before the game what the game is about and so they could not have prepared any scheme in advance. All they are allowed to do is that after each time the clock rings, either of them can guess the other person's number, or both of them can stay silent. The game ends when the first guess has been made.

If any one of them guesses the other person's number correctly, then both of them win €1 million, and they lose if the guess is incorrect.

How can Ali and Barb best play this game to win? Assume that both of them know that the other person is perfect at logical reasoning, and clever enough to do the right thing.