Discrete Mathematics I — So 18
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Exercise Sheet 1

Due date: April 24th at 4:15 PM

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 We have a chess tournament with $2n$ players and in the first round, we want to pair them up on n tables. In how many ways can the first round be played in the following cases.

(a) All tables are identical and it does matter which player gets which colour.

(b) All tables are distinct and it doesn't matter which player gets which colour.

(c) There is one special table[1], which is important to everyone, and the other tables are identical. Also assume that it does matter which player gets which colour.

For each case write down explicitly the set that you are counting.

Exercise 2 Recall that for $n \in \mathbb{N}$, $D(n)$ is the set of divisors of n, and $d(n) = |D(n)|$.

(a) Find an exact expression for $d(n)$ in terms of the prime factorisation of n.

(b) Let $s(n) = \sum_{d \in D(n)} d$ be the *sum* of all of the divisors of n. Find an expression for $s(n)$ in terms of its prime factorisation.

Exercise 3 How many ways are there to travel in the 3-dimensional Euclidean space from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by taking steps one unit in the positive x direction or one unit in the positive y direction or one unit in the positive z direction? (Moving in the negative directions is prohibited.)

[1] with Pretzels!

Exercise 4

(a) How many $n \times n$ matrices with entries from $\{0, 1, \ldots, q-1\}$ are there?

(b) Let q be a prime number. How many non singular matrices are there over the field with q elements? (In other words: How many matrices from question (a) have a determinant that is not divisable by q?)

For (b), recall that when q is a prime number, the finite field of q elements is just the residue classes of integers modulo q. For example, for $q = 2$ we have the field $(\{0, 1\}, +, *)$ where $0 + 0 = 1 + 1 = 0$, $0 + 1 = 1 + 0 = 1$, $0 * 1 = 1 * 0 = 0 * 0 = 0$ and $1 * 1 = 1$. The notions of linear independence/dependence of vectors over finite fields work exactly the same way as they did for the fields $\mathbb{R}$ and $\mathbb{C}$.