Discrete Mathematics I So 18
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Exercise Sheet 11

Due date: July 10th at 4:15 PM

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 A 3-factor in a graph G is a 3-regular spanning subgraph of G. Prove that every $(2k+1)$-regular graph on $4k$ vertices has a 3-factor.

Exercise 2 Prove or disprove that every graph G with $\chi(G) = k$ has some proper k-coloring in which some color class has $\alpha(G)$ vertices.

Exercise 3 Let G be an n vertex graph, $\alpha(G)$ its independence number, $\beta(G)$ the vertex cover number, $\alpha'(G)$ the matching number, $\beta'(G)$ the edge cover number [1]. Prove that

(1) $\alpha(G) + \beta(G) = n$;

(2) $\alpha'(G) + \beta'(G) = n$.

Exercise 4 The university wants to schedule one-hour final exams for the following courses: Algebra, DMI, Logic, Advanced Breath Holding (ABH), Can you Fry That (CFT), History of Ice Cream (HIC), and Baby Talk (BT). Due to the renovation of their main exam facility, the university must rent an outside building (which actually has several exam rooms). The rent is by the hour, so instead of just scheduling the exams separately, the university would like to schedule some exams parallel in order to save money.

The following pairs of courses do not have any students in common, and hence can be scheduled parallel: (Algebra, ABH), (Algebra, CFT), (Algebra, HIC), (DMI, ABH), (DMI, HIC), (DMI, BT), (ABH, HIC), (ABH, BT), (CFT, BT). Find the minimum number of time slots required to schedule the exams without any conflicts.

[Hint (to be read backwards): .melborp gniroloc hparg a sa ti ledoM]

Exercise 5 Prove that the complement of a bipartite graph is perfect (without using the perfect graph theorem).

[Hint (to be read backwards): .sesicrexe suoiverp eht morf stluser eht esu nac uoY]

[1] minimum number of edges that cover the vertex set