Discrete Mathematics I So 18
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Exercise Sheet 13

Due date: Aug 1st at 4:15 PM

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 A graph G is called a minimal non-planar graph if G is non-planar and each subgraph of G is planar.

(1) Prove that a minimal non-planar graph is 2-connected.

(2) Prove that if G is a non-planar graph and there is a separating set $S = \{x, y\}$ then there is an S-lobe which together with the edge xy is also non-planar.

Exercise 2 Recall from the previous exercise sheet that a planar graph G is outerplanar if there is an embedding of it in the plane such that all vertices are on the boundary of the outer face. What is the largest number of edges in a simple outerplanar graph?

Exercise 3 Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than 6.

Exercise 4 Show that one can draw a K_7 and a $K_{3,3}$ on a torus without any crossings. [1]

Exercise 5 A plane graph is self-dual if it is isomorphic to its dual.

(1) Prove that if G is a self-dual plane graph on e edges and v vertices then $e = 2v - 2$

(2) Find a self-dual plane graph of on v vertices for each $v \geq 4$.

[1] It might help to look at the construction of the torus using a unit square in $\mathbb{R}^2$ where you identify the points $(0, x)$ and $(1, x)$, and the points $(y, 0)$ and $(y, 1)$.