FREIE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK
DISCRETE MATHEMATICS I

SUMMER SEMESTER 2018
JUNE 14TH, 2018

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EXERCISE	1.	2.	3.	4.	5.	6.
POINTS						

PRACTICE EXAM

Show all your work and state precisely the theorems you are using from the lecture. Ideally, try to solve the sheet within a time limit of 180 minutes, without using any books, notes, etc ... (but of course this is not mandatory if you feel it would not yet make sense this way). It will be graded like the Final Exam, but the points do not count towards your exercise credit.

Exercise 1 [10 points]

(1) State the definition of the Ramsey number $R(k)$.

(2) Prove that $R(4) \leq 32$.

[A prospective exercise from a future homework sheet for the curious ones (not part of the practice exam, do not submit it now): Prove that $R(4) \leq 20$. (Any improvement between 32 and 20 will be rewarded proportionally.) Bonus: Prove that actually $R(4) \leq 18$.]

Exercise 2 [10 points]
Prove that among a group of n people, for any integer $n \geq 2$, there will always be two people who know exactly the same number of people (assuming that knowing is a mutual relationship).

Exercise 3 [10 points]
Let G be a graph which is k-regular, i.e., every vertex has degree k. Prove that if there no cycles of length 3 and 4 in G[1], then G must have at least $k^2 + 1$ vertices.

Exercise 4 [10 points]

(1) Define a poset and an antichain in a poset.

(2) Prove that in the boolean poset $(2^{[n]}, \subseteq)$ the largest antichain has size $\binom{n}{\lfloor n/2 \rfloor}$.

[1] The Petersen graph that you saw in the lectures is an example of such a graph with $k = 3$.

Exercise 5 [10 points]

(1) State the definition of the Striling number of second kind.

(2) Let $S(n, r)$ be the Stirling number of the second kind. Prove that

$$r!S(n, r) = \sum_{i=0}^{r} (-1)^{r-i} \binom{r}{i} i^n.$$

Exercise 6 [10 points]

For $n \in \mathbb{N}$, let p_n denote the number of permutations of $[n]$ whose cycle decomposition only contains cycles of length at most 2.

(1) Find a recurrence relation for the sequence $(p_n)_{n \geq 0}$.

(2) By solving this recurrence, obtain an explicit formula for p_n.