Exercise Sheet 1

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Discrete Mathematics II, Winter 2011/12
Due date: October 25th (Tuesday) by 10:00, at the end of the lecture.

Problem 1 Give two proofs that the Petersen graph is nonplanar.

(*a*) Using Kuratowski's Theorem.

(*b*) Using Euler's Formula.

Problem 2 Prove that every n-vertex plane graph isomorphic to its dual has $2n-2$ edges.
For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

Problem 3 Given a plane graph G, draw the dual graph G^* so that each dual edge intersects its corresponding edge in G and no other edge. Prove the following.

(*a*) G^* is connected

(*b*) If G is connected, then each face of G^* contains exactly one vertex of G.

(*c*) $(G^*)^* = G$ if and only if G is connected.

Problem 4 Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6.
For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than 6.

Problem 5 Prove that every 3-connected graph on at least six vertices that contains a subdivision of K_5, also contains a subdivision of $K_{3,3}$.