Exercise Sheet 10

Tibor Szabó
Discrete Mathematics II, Winter 2011/12
Due date: January 17th (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1 Prove that $R(3,4) = 9$ and $R(3,5) = 14$. (*Hint:* To construct a coloring for the later case, you could try to label the vertices with the elements of $\mathbb{F}_{13}$.)

Problem 2 Prove that the Paley-coloring on 17 vertices does not contain a m.c. K_4.
(In the Paley coloring the vertices are labeled with the elements of the p-element field $\mathbb{F}_p$. An edge xy is colored with red if $x - y \in Q_p$, otherwise the edge is colored blue. Here $Q_p = \{z^2 : z \in \mathbb{F}_p\}$ is the set of *quadratic residues* modulo p.) (*Hint:* You should try to prove and use the many symmetries of the Paley coloring.)

Problem 3. Let $R_r(3)$ be the generalization of the Ramsey number $R(3,3)$ to r colors.
(a) Come up with the definition of $R_r(3)$
(b) Prove that $R_r(3)$ is finite
(c) Show that $R_r(3) \leq \lfloor e \cdot r! \rfloor + 1$

Problem 4. Prove that if the first $\lfloor k!e \rfloor$ integers are colored with k colors, then there are three (not necessarily distinct) integers x, y, z having the same colors which satisfy $x + y = z$.

Remark. This is a van der Waerden type theorem for a monochromatic solution of a linear equation, only the equation is different. In van der Waerden's Theorem we consider $x+y = 2z$, while here we consider $x+y = z$. The difference seems small, still this problem is more of an exercise than van der Waerden's Theorem.

Problem 5. Prove that for every k there exists an integer $n(k)$ such that if you color the *subsets* of an $n(k)$ element set V with k colors, then there are two disjoint non-empty subsets X, Y such that X, Y, and $X \cup Y$ all have the same color.