Exercise Sheet 13

Tibor Szabó
Discrete Mathematics II, Winter 2011/12
Due date: February 7th (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1. Improve the bound from previous week on the chromatic number of the unit distance graph and prove that $\chi(G_n) \leq 9^n$.

(*Hint:* As a first step, select a maximal set $S \subseteq \mathbb{R}^n$ of points such that the distance between any two of them is at least $1/2$. Color these points such that no two of them is colored the same if their distance is at at most 2.)

Problem 2 A tournament is a complete graph where each edge is oriented one way. (Formally: D is a tournament with vertex set $V(D)$ and edge set $E(D)$ if for every $\{x, y\} \in \binom{V(D)}{2}$ either $(x, y) \in E(D)$ or $(y, x) \in E(D)$.)
A tournament D is *transitive* if there is an ordering on its vertices $v_1, \ldots, v_n$ such that $(v_i, v_j) \in E(D)$ if and only if $i < j$.
(a) Prove that in any tournament on 2^k vertices there is a transitive subtournament of order k.
(b) Prove that there exists a tournament on $\sqrt{2}^k$ vertices with no transitive subtournament of order k.

Problem 3. Let $G = (V, E)$ be a bipartite graph on n vertices. Prove that $\chi_l(G) \leq \log_2 n$.

Problem 4. Suppose $n \geq 4$ and let H be an n-uniform hypergraph with at most $\frac{4^{n-1}}{3^n}$ edges. Prove that there is a coloring of the vertices of H by 4 colors so that in every edge all 4 colors are represented.