Exercise Sheet 2

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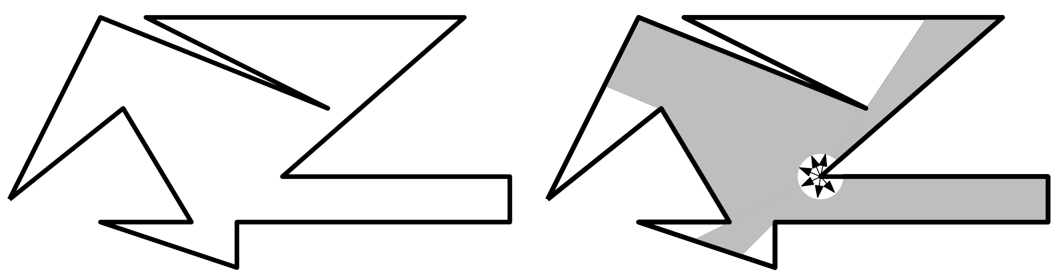
Discrete Mathematics II, Winter 2011/12
Due date: November 1st (Tuesday) by 10:00, at the end of the lecture.

Problem 1 Use Kuratowski's Theorem to show that a graph is outerplanar if and only if it does not contain a subdivision of K_4 or $K_{2,3}$.

Problem 2 Prove, without using the Four Color Theorem, that every outerplanar graph is 3-colorable.

Problem 3 Apply Problem 2 to prove the Art Gallery Theorem: If an art gallery is laid out as a simple polygon with n sides, then it is possible to place $\lfloor n/3 \rfloor$ guards such that every point of the interior is visible by some guard. Construct a polygon that does require $\lfloor n/3 \rfloor$ guards.



An art gallery and what a guard sees from a corner

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Problem 4 Prove Wagner's Theorem: A graph is planar if and only if it does not contain K_5 or $K_{3,3}$ as a minor.

Problem 5

(a) Give a drawing of K_6 in the real projective plane without any crossing. (Think of the projective plane as a closed disc where opposite points of the boundary circle are identified.)

(b) Give a drawing of K_7 on the torus without any crossing. (Think of the torus as the unit square $[0,1]^2$, where each boundary point $(0,y)$ is identified with $(1,y)$ and point $(x,0)$ is identified with $(x,1)$.)

Problem 6 (*) Prove that a maximal planar graph is 3-colorable if and only if it is Eulerian. (Hint: For sufficiency, use induction on $n(G)$. Choose an appropriate pair or triple of adjacent vertices to replace with appropriate edges.)