Exercise Sheet 3

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Discrete Mathematics II, Winter 2011/12
Due date: November 8th (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1 Determine the crossing numbers of $K_{2,2,2}, K_{1,2,2,2}$ and $K_{2,2,2,2}$.

Problem 2. It is conjectured that $\nu(K_{m,n}) = \lfloor \frac{m}{2} \rfloor \cdot \lfloor \frac{m-1}{2} \rfloor \cdot \lfloor \frac{n}{2} \rfloor \cdot \lfloor \frac{n-1}{2} \rfloor$. Suppose that this conjecture holds for $K_{m,n}$ and m is odd. Prove that the conjecture then holds also for $K_{m+1,n}$.

Problem 3. Prove that $\nu(K_n) \geq \frac{1}{80} n^4 + O(n^3)$, using the fact that the conjecture in Problem 2 about the crossing number of $K_{6,n-6}$ is known to be true.

Problem 4 Prove that a 3-regular simple graph has a perfect matching if and only if its edge-set can be decomposed into P_4s.

Problem 5 Exhibit a maximum matching in the graph below and give a short proof that it has no larger matching.