Exercise Sheet 4

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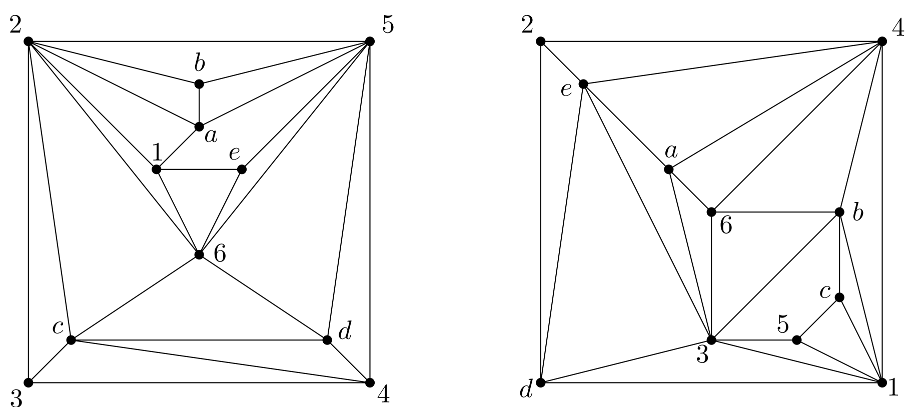
Discrete Mathematics II, Winter 2011/12
Due date: November 15th (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1 Prove that a tree T has a perfect matching if and only if $o(T-v) = 1$ for every vertex $v \in V(T)$.

Problem 2 Prove the Weak Perfect Graph Theorem for bipartite graphs.

Problem 3 The *thickness* of a graph G is the minimum number k, such that G can be decomposed (that is, the edge set can be partitioned) into k planar graphs. In this problem we are concerned with the maximum chromatic number a graph with thickness 2 can have and establish the best known bounds.
(i) Show that a graph with thickness 2 can be colored properly with 12 colors.
(ii) Prove that a graph with thickness 2 can have chromatic number as large as 9.
(*Hint: Stare at the the picture for a while.*)



Problem 4. Prove that $\chi(G) + \chi(\bar{G}) \le n(G) + 1$ holds for every graph G. Give an example for every $n(G)$ to show that the statement is "best possible", i.e., equality can hold. (For extra credit: Can you give two non-isomorphic examples for every (large enough) $n(G)$?)

Problem 5. (*) (i) Suppose that for an integer $k \in \mathbb{N}$ and simple graph G we have that $\delta(G) \ge k$ and $n(G) \ge 2k$. Prove that $\alpha'(G) \ge k$.
(ii) Conclude that if $n(G)$ is even and $\delta(G) \ge n(G)/2$, then G has a perfect matching. Show that the minimum degree condition guaranteeing that G has a perfect matching is best possible.