Exercise Sheet 5

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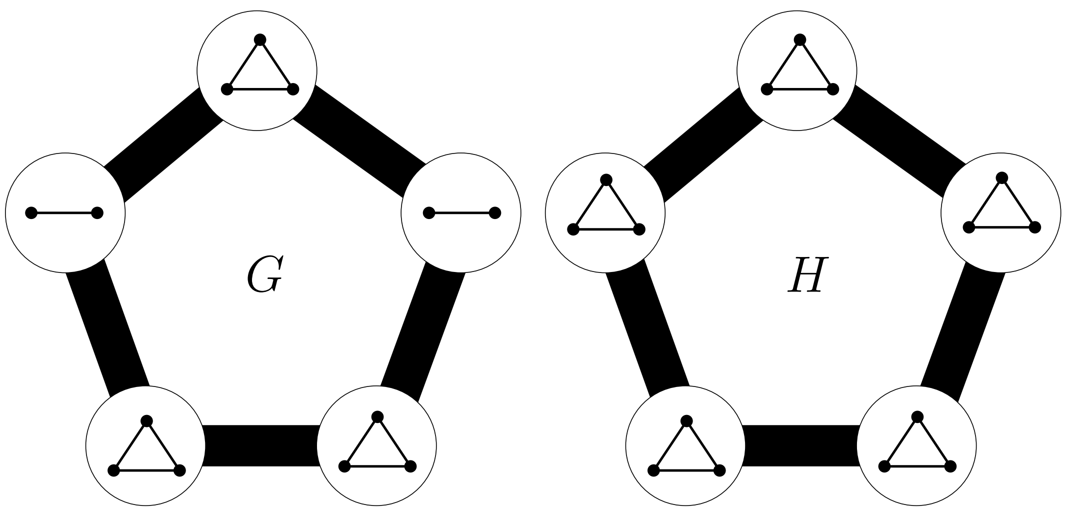
Discrete Mathematics II, Winter 2011/12
Due date: November 22nd (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1. Prove that if G is color-critical then $M(G)$ (the Mycielski of G) is also color-critical.

Problem 2. Disprove the Hajós Conjecture for $k = 7$ and $k = 8$.
Prove that $\chi(G) = 7$ but G has no K_7-subdivision. Prove that $\chi(H) = 8$ but H has no K_8-subdivision. (Thick edges below indicate that every vertex in one circle is adjacent to every vertex in the other.)



Problem 3. Use Brooks' Theorem to prove Vizing's Theorem for graphs with maximum degree 3.

Problem 4. Let G be a regular graph with a cut-vertex. Prove that $\chi'(G) = \Delta(G) + 1$.

Problem 5. Prove that for every simple bipartite graph G there is a $\Delta(G)$-regular simple bipartite graph H that contains G. (In the lecture we proved the analogous statement for multigraphs.)