Exercise Sheet 7

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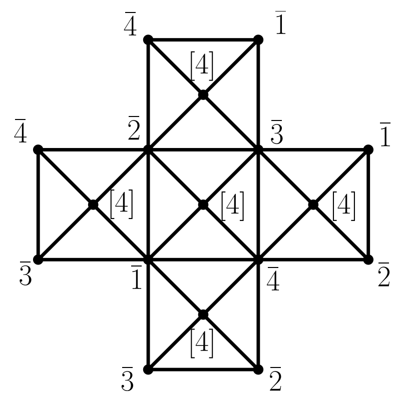
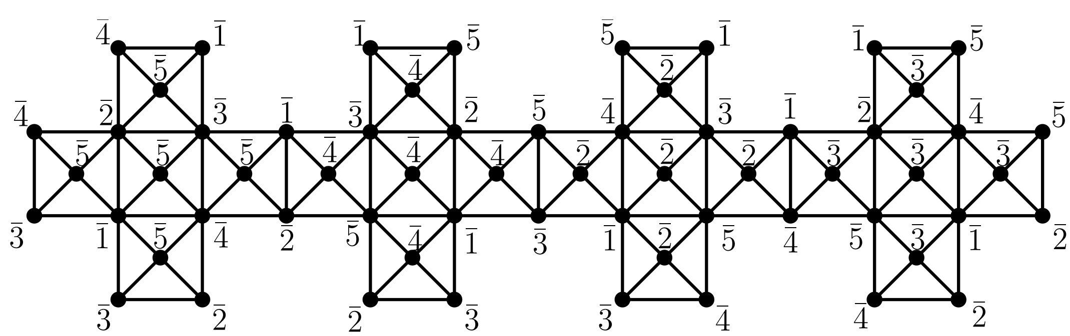
Discrete Mathematics II, Winter 2011/12
Due date: December 6th (Tuesday) by 12:30, at the beginning of the exercise session.

Problem 1. Prove that $K_{k,m}$ is k-choosable if and only if $m < k^k$.

Problem 2. Show that the list-chromatic number of a planar graph can be as large as 5.
(a) Show that the following graph has no proper coloring from the assigned lists. (In this part of the exercise $\bar{i}$ denotes the set $[4] \setminus \{i\}$.)



(b) Let G be the graph obtained from the graph below by adding an extra vertex to the outside face and connecting it to all vertices on the boundary. Show that if the new vertex is assigned the list $\bar{1}$, then there is no proper list coloring of G from the assigned lists. (In this part of the exercise $\bar{i}$ denotes the set $[5] \setminus \{i\}$.)



Problem 3. A mouse eats its way through a $3 \times 3 \times 3$ cube of cheese by eating all the $1 \times 1 \times 1$ subcubes.

If it starts at a corner subcube and always moves on to an adjacent subcube (sharing a face of area 1), can it do this and eat the center cube last? (Ignore gravity...)

Problem 4. On a chessboard, a *knight* can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate. Prove that no $4 \times n$ chessboard has a *knight's tour*: a traversal by knight's moves that visits each square once and returns to the start.

Problem 5. Prove that an n-vertex graph with at least $\binom{n-1}{2} + 2$ edges contains a Hamilton cycle.
Conclude the value of $ex(n, C_n)$ for every n.